THE FRINGE OF THE ATMOSPHERE AND THE ULTRA-VIOLET LIGHT THEORY OF AURORA AND MAGNETIC DISTURBANCES*

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ABSTRACT. An atmosphere in isothermal equilibrium has no natural limit, an atmosphere in adiabatic equilibrium has, however, one. Beyond this "inner" adiabatic atmosphere lies the outer atmosphere formed by molecules "evaporated" from the outer surface of the "inner" atmosphere. The fringe region lies at the farther limits of this outer atmosphere, where the molecules moving without any collision describe orbits of enormous dimensions.

Taking atomic oxygen and molecular nitrogen as the upper atmosphere constituents the height of the critical level or level of negligible collision is calculated by the authors and is found to be 770 km. Above 930 km. there is practically no collision between the atoms.

Ionization of the atoms in the fringe region by solar ultra-violet radiation will diminish the rate of escape owing to their entanglement with the earth's magnetic field. Critical velocities of escape of oxygen ions from different latitudes of the earth are calculated.

The increase with height of the viscous drag of the earth measured by kinematical viscosity η/ρ is taken into account and it is shown that the atmosphere rotates with the earth so long as there is appreciable collision between atoms.

Molecular densities at different levels of the fringe region are estimated. The merging of the fringe region into interstellar space takes place between 2000 and 2500 km. above the earth's surface.

Existence of super-elastic collisions between an excited (metastable) atom and a neutral particle extends the fringe region. Of the various metastable states of the constituent particles only those of oxygen atoms are capable of producing any effect by super-elastic collisions. It is found that roughly about 10^4 atoms in the ^1S state and 10^6 atoms in the ^1D state suffer collisions per second with other atoms. The atoms in the ^1S state can shoot a neutral oxygen atom to a height of about 14,000 km. above earth's centre while an atom in the ^1D state can shoot it to a height of about 9,500 km. The density distribution in the fringe region and the merging with the interstellar space (which takes place at about 2500 km. above the earth' surface) are found to be only slightly modified by the presence of these high speed particles.

By considering the motion of ions in the earth's magnetic and gravitational fields, taking into account the variations of g and H with altitude and also considering the rotation of the earth magnet it is found that an ion starting at the equator at 12 o'clock noon at 40,000 km. height

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and descending along the magnetic line of force with a velocity of 1 km /sec will enter the earth's atmosphere at 12-54 local hour. Bearing of the above results on the Ultra-violet light theory of Aurorae and magnetic storms as suggested by Hulburt (the essential feature of which is the distillation of speedy ionized particles from lower to higher latitudes by the earth's magnetic field) is discussed.

If it is assumed that the high speed atoms are ionized at a height of about 40,000 km. the time of descent as calculated above by the authors agrees with that calculated by Hulburt. Analysis shows, however, that by no known means particles from lower levels can reach 40,000 km level to which they must rise in order to reach the auroral latitude Again according to Hulburt the particles driven upwards by super-elastic collisions will be ionized in three hours Recent calculations on atomic absorption coefficient of oxygen shows, however, that the time required should be several orders higher than that assumed by Hulburt.

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PART I

Fringe of the Atmosphere and its Extension

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LIST OF SYMBOLS

a—Earth's radius.

A (n, l)—A constant, depending on the orbit in which the electron is captured.

C— $\sqrt{\dfrac{M}{HR}}$.

c—Velocity of light (Part II, §2).

c, c'—Velocities of two colliding gas molecules (Part I, §3).

E—Energy required to ionize an atom (Part II, §5); Intensity of the electric field generated by the rotation of the magnetic lines of force of the earth (Part II, §4).

E_r, E_θ—Components of the electric field E,

e—Charge on an ion.

g—Acceleration due to gravity at a distance r from the centre of the earth.

g_0—Acceleration due to gravity at the surface of the earth.

H—Intensity of the earth's magnatic field at a distance from the centre of the earth.

H_r, H_θ—Components of H.

H_0—Intensity of the earth's magnetic field at the surface of the earth.

h—Planck's constant.

$I(\nu)_0$—Intensity of solar radiation of frequency ν outside the earth's atmosphere.

k—Boltzmann's constant.

M—Magnetic moment of the earth.

m—Molecular or atomic weight (Part I, §3), weight of an ion (Part II).

m_1, m_2—Molecular or atomic weights.

N—Total number of atoms in a column of 1 sq. cm. cross-section above the critical level.

n—Molecular density.

n_0—Molecular density at the datum level.

p—Pressure at any particular level.

p_0—Pressure at the datum level.

q—Number of ions formed per sec. in a column of 1 sq. cm. cross-section above the critical level.

q_+—Number of positive ions in the column.

q_-—Number of electrons in the column.

q_0—$\dfrac{mg}{KT} \cdot r_0$.

q'_0—$\dfrac{mg}{KT} \cdot r_0 - 2$.

q—Number of ions recombining per second in a column of 1 sq. cm. cross-section above the critical level.

$q(n, l)$—A constant, depending on the orbit of capture.

R—Radius of curvature of the helical path of an ion.

r—Radius vector (distance from the centre of the earth).

r_0—Distance of the datum level from the centre of the earth.

T—Temperature of the gases of the atmosphere (in absolute degrees).

T_s—Temperature of the sun (in absolute degrees).

t—Time (in seconds).

u_g—Gravitational magnetic drift velocity of an ion.

V—Relative velocity of two colliding gas molecules (Part I, §3), Relative velocity of an ion ($=v-u_g$) (Part I, §2).
Energy in volts (Part I, §6).

v—Velocity of an atom or molecule (Part I, §3).
Velocity of an ion (Part II, §§2, 3, 4).

v'—Electromagnetic drift velocity.

Z—Effective atomic number of an ion.

z—Height above the datum level.

α—Recombination coefficient of ions and electrons.

$4\pi\beta$—Solid angle subtended by the sun at the earth.

γ—Ratio of specific heat at constant pressure to the specific heat at constant volume (Part I, §2): a constant (Part I, §4).

γ_1—A constant.

η—Coefficient of viscosity of a gas.

θ—Angular co-ordinate (Part I, §3); Angle made by r with the axis of the earth (Part II, §4).

θ_0—Co-latitude of the place where a particular magnetic line of force terminates on the earth's surface.

λ_0—Wavelength of radiation corresponding to frequency ν_0.

ν—Frequency of radiation.

ν_0—Threshold frequency for dissociation or ionization of an atom.

ρ_1, ρ_2, etc.—Densities of gases at a particular level of the atmosphere.

$(\rho_1)_0$, $(\rho_2)_0$—Densities of gases at the datum level.

σ—Molecular diameter.

$\tau(\nu)$—Absorption per atom of radiation of frequency ν.

τ_0—Limiting value of $\tau(\nu)$ at $\nu=\nu_0$.

ϕ—Angular co-ordinate (Part I, §3). Total angular shift of an ion path due to electromagnetic drift. (Part II, §4).

χ_1—Angular measure (in degrees from the poles) of the region of escape of ions from the earth's atmosphere.

$\psi(\nu)$—Ionization probability.

ω—Angular velocity in the helical motion of an ion.

Ω—Angular velocity of rotation of the earth.

§1 INTRODUCTION

It is now conventional to divide the atmosphere into three regions—the Lower, the Middle and the Upper Atmosphere. The lower atmosphere, which is the same as the Troposphere, extends from the ground level to a height of 10 to 15 km.; the middle atmosphere, from 15 to 100 km., includes the Stratosphere lying immediately above the Troposphere and also the Ozonosphere between 25 and 45 km. The atmosphere from 100 km. above is called the Upper Atmosphere and is more or less ionized. Besides these three regions, one should also consider the "fringe" region of the upper atmosphere which is supposed to exist beyond the so-called limit of the upper atmosphere. Attention to this "fringe" region of the atmosphere was first drawn by Johnstone Stoney. [1] The fringe has recently awakened new interest owing to its being a possible source of speedy charged particles which may be the cause of magnetic storms, aurorae, etc.[2] The purpose of the present paper is to make a critical study of the origin and properties of the fringe region (Part I) and to examine how far the charged particles originating in it by the action of the ultra-violet rays of the sun may be a contributory cause of the above-named geophysical phenomena (Part II).

PART I. FRINGE OF THE ATMOSPHERE AND ITS EXTENSION

§2. ATMOSPHERE IN ISOTHERMAL AND IN ADIABATIC EQUILIBRIUM; NATURAL LIMIT OF THE ATMOSPHERE

Let us consider the idealised case of a gaseous atmosphere above a non-rotating earth undisturbed by solar rays. Such an atmosphere, if left to itself, would, after a sufficient time, attain a state of isothermal equilibrium. Under equilibrium condition, the constituent gases at different heights would be distributed according to the law

$$\left.\begin{aligned} \rho_1 &= (\rho_1)_0\, e^{-\frac{m_1 g}{kT}\cdot z} \\ \rho_2 &= (\rho_2)_0\, e^{-\frac{m_2 g}{kT}\cdot z} \\ &\text{etc.} \quad \text{etc.} \end{aligned}\right\} \qquad \ldots \quad (1)$$

where

ρ_1, ρ_2, etc. are the densities of the different gases at a height z above the earth's surface;

$(\rho_1)_0$, $(\rho_2)_0$, etc. are the densities of the different gases at the surface of the earth ;
m_1, m_2, etc. are the molecular weights of the constituent gases ;
T is the constant absolute temperature of the atmosphere.

It is easily seen from eqn. (1) that the density of the atmosphere diminishes exponentially so that ρ attains zero value only at infinity. Such an atmosphere, as long as it obeys gas kinetic laws, will not have any natural limit.

Secondly, we may consider the idealised case of the opposite extreme in which the whole mass of the atmosphere is subject to turbulence (caused by heating, etc.) and possesses convective motions throughout its entire height. Under such circumstances the atmosphere may be said to be in adiabatic equilibrium as opposed to isothermal equilibrium. The adiabatic equilibrium is brought about by the movement of masses of air from lower to higher level and *vice versa*, without any loss or gain of heat.

From the general equation of equilibrium of the atmosphere

$$\frac{\partial p}{\partial z} = -g\rho$$

we get

$$k\gamma\rho^{\gamma-1}\frac{\partial \rho}{\partial z} = -gz$$

by applying the adiabatic law $p = k\rho^{\gamma}$.

Integrating we get

$$\frac{k\gamma}{\gamma-1}(\rho_0{}^{\gamma-1} - \rho^{\gamma-1}) = gz \qquad \ldots \quad (2)$$

where ρ_0 is the density of the gas at the surface of the earth ;
γ is the ratio of the specific heat at constant pressure to the specific heat at constant volume.

Thus equation (2) expresses the law according to which density and temperature fall off with height. In contrast with equation (1) we find that ρ becomes zero at a finite value of z given by

$$\left.\begin{aligned} z &= \frac{k\gamma\rho_0{}^{\gamma-1}}{g(\gamma-1)} \\ &= \frac{p_0\gamma}{g\rho_0(\gamma-1)} \end{aligned}\right\} \qquad \ldots \quad (3)$$

We thus see that in the case of an atmosphere in adiabatic equilibrium we can conceive of a natural limit. If, for instance, we assume that the entire terrestrial atmosphere is in adiabatic equilibrium, its natural limit can easily be calculated and is found to lie at a height of about 29 km.,[3] above the surface of the earth ($\rho_0 = \cdot 001205\ gm/cm^3$, $\gamma = 1{\cdot}4$, $p_0 = 760\ mm.$).

Further considerations show that the surface at the outer limit of the atmosphere—though a natural upper boundary of the atmosphere in the lower region in adiabatic equilibrium—will not be a surface separating a region of perfect vacuum above and a region containing gas molecules below. Molecules from the adiabatic atmosphere below will be constantly evaporating, as it were, due to thermal agitation, across the surface of separation to the vacuous space above in much the same way as liquid molecules escape from the body of a liquid to the space above its surface. Thus the region above the natural limit will contain molecules and this region may be called the " outer atmosphere." If we assume that there is appreciable, though small, collisions between the molecules, then these will be distributed according to equation (4) provided the temperature is taken to be constant.

$$\left.\begin{aligned} \rho_1 &= (\rho_1)_0 e^{-\frac{m_1 g}{kT} a\left(\frac{z}{a+z}\right)} \\ \rho_2 &= (\rho_2)_0 e^{-\frac{m_2 g}{kT} a\left(\frac{z}{a+z}\right)} \\ &\text{etc.} \qquad \text{etc.} \end{aligned}\right\} \quad \ldots \quad (4)$$

The outlying region of this " outer atmosphere," in which collisions are few and far between, may be called the spray or fringe of the atmosphere. The level at which the transition from the outer atmosphere to the spray region occurs will, of course, depend on the temperature, pressure and other factors of the " outer atmosphere."

In this fringe region of the atmosphere, molecules will move freely with the velocity acquired at the last collision in the lower region and subject only to the pull of gravity will describe parabolic, elliptic or hyperbolic paths according to the magnitude and direction of their velocities. Molecules with hyperbolic orbits will, of course, escape from the earth and the velocity necessary for such escape will be given by

$$v^2 \geq \frac{2ga^2}{r}$$

where v = velocity of the molecule ;
g = acceleration due to gravity at the level from which the molecule escapes ;
a = the earth's radius ;
r = distance of the level of escape from the centre of the earth.

If the idealised conditions described above were applicable to the earth, then we would have the following picture of the atmosphere: The constituent gases are thoroughly mixed and the atmosphere is in adiabatic equilibrium up to a height of about 29 km. which is the natural limit of the atmosphere with lapse rate of about 10° per km. This region in which the temperature falls linearly with height is the "inner atmosphere." Beyond this is the "outer atmosphere" formed of molecules evaporated from the inner atmosphere. The outer atmosphere is in isothermal equilibrium and its outlying regions form the spray of the atmosphere where there is practically no collision and the molecules describe parabolic, elliptic or hyperbolic paths round the earth. As mentioned in the Introduction, it is this last named region of the atmosphere which will form the subject matter of our study in the following sections

§3. CRITICAL LEVEL

The picture of the ideal adiabatic atmosphere given above is unfortunately widely different from the actual condition of the atmosphere. The lapse rate for instance is only about 5°C/km. instead of about 10°C/km. Again if the loss and gain of heat by radiation and absorption by each element of the atmosphere for a permanent atmospheric arrangement be considered, it can be shown that on the assumption of a uniform constitution of the atmosphere, the adiabatic state could not extend to a height greater than given by $p=\frac{1}{2}p_o$, where p_o is the surface pressure. If the atmosphere be not uniform the height of the adiabatic layer increases and for the actual constitution of the atmosphere (containing varying amount of water vapour at different heights), it has been shown by Gold [4] that the height of the adiabatic layer should lie between $p=\frac{1}{2}p_o$ and $p=\frac{1}{4}p_o$ (*i.e.*, between $z=5\frac{1}{2}$ km. and $z=10\frac{1}{2}$ km.). Above this adiabatic layer we have the outer atmosphere and here also recent observations show that the condition is far from isothermal, due principally to the photo-chemical and photo-ionizing action of the solar rays. The outer atmosphere, the base of which is approximately isothermal and is to be identified with the stratosphere, thus extends to great heights, and it is near the outside (boundary) of this region that we shall look for the "fringe" region of the outer atmosphere. The collisional frequency in this region is negligible and the molecules have either a chance to escape or to describe closed paths round the earth or to fall back to the earth after describing parabolic orbits and reaching enormous heights from the earth's surface.

The first attempt to calculate the height at which collisions begin to have negligible effect was made by Milne [5] and was later followed up by J. E. Jones.[6] It has been shown by the latter that the failure of previous workers to estimate the height of the "ceiling of the atmosphere" or the level above which there is very little chance of a collision is due to their assumption of uniform molecular density along the free path of a molecule. At high altitudes of the atmosphere the free path of a molecule is appreciably large and the molecular density also decreases rapidly upwards. Along the free path of a molecule, therefore, the molecular density is liable to change appreciably. Thus account should be taken of the fact that the probability of collision of a molecule is a function not only of its velocity but also of its origin and direction of motion, since the molecular density will vary in different manners in different directions.

Milne and Jones start with the well-known expression of Tait for the probability of collision of a molecule of given velocity c—

$$\theta(c) = n\sigma^2 \sqrt{\frac{m^3}{8\pi k^3 T^3}} \int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} e^{-\frac{mc'^2}{2kT}} c'^2 . V . dc . \sin\theta \, d\theta . \, d\phi .$$

where n = molecular density ;
σ = molecular diameter ;
m = molecular weight ;
c, c' = velocities of the colliding molecules ;
V = relative velocity between the colliding molecules and is given by $V^2 = c^2 + c'^2 - 2cc' \cos\theta$;
θ, ϕ = angular co-ordinates of the direction of motion.

The expression can be put in the form :—

$$\theta(c) = \frac{\sqrt{\pi} . n\sigma^2}{c\frac{m}{2kT}} \quad \psi\left(c\sqrt{\frac{m}{2kT}}\right)$$

where ψ is a function of the form $\psi(x) = xe^{-x^2} + (2x^2+1)\int_0^x e^{-y^2} dy$ ... (5)

In order to evaluate the above equation, we require a knowledge of the various constants n, σ, etc. n in particular varies with height and the usual expression for its variation is given in eqn. (1). When we are considering the conditions at great heights we have got to take into account the change in the pull of gravity as well as the attraction of the mass of air which varies with height, as one goes outwards. Taking these factors into consideration Milne shows that a more correct expression for n at great heights in the outer atmosphere, assuming a constant temperature, is given by—

$$n = n_0 \left(\frac{r_0}{r}\right)^2 e^{-q_0'\left(1 - \frac{r_0}{r}\right)} \qquad \ldots \quad (6)$$

where n_0 = molecular density at a level at a distance r_0 from the earth ;
n = molecular density at the level at a distance r where collision is being considered ;

$$q_o' = \frac{mg}{k'T} \cdot r_o - 2 = q_o - 2.$$

The estimate of the height of the " critical level " is now made by the help of equations (5) and (6) and introducing the idea of the " cone of escape." Let us suppose that an observer ascends gradually upwards from a level where molecular density is small but appreciable. If the molecules were opaque, then the hemispherical sky above the observer would appear to him absolutely opaque, a line drawn from the observer towards any direction would pass through many molecules, one behind the other. If the observer continues his ascent, he will reach a level where the molecules overhead will gradually thin away and will ultimately just fill the sky, *i.e.*, a line drawn upwards will pass through only one molecule whereas a line drawn in any other direction will still pass through more than one molecule. Mounting still higher, he will find his sky overhead gradually clearing up and he will " see " a cone with its axis vertical within which his sky will be clear. It is obvious that this cone will open out with height and the observer will finally have his whole sky clear. This cone, the axis of which is vertical and the angle of which increases with height, is the " cone of escape " of the molecules. A molecule moving within the solid angle of this cone will have some chance of escaping without a collision. θ, the semi-vertical angle of the cone of escape, is shown by Jones to be related to r and c by the equation

$$e^{q_o' \frac{r_o}{r}} - 1 = \frac{q_o' e^{q_o'} \cos\theta \, c^2 m}{2k'T \sqrt{\pi} \, n_o \sigma^2 r_o \psi\left(c\sqrt{\frac{m}{2k'T}}\right)} \, . \qquad \ldots \quad (7)$$

The lowest estimate of the level above which a molecule may escape without a collision is easily obtained from eqn. (7) by putting $\theta = o$ and $c = \infty$.

Thus

$$e^{q_o' \frac{r_o}{r_c}} - 1 = \frac{q_o' e^{q_o'}}{\pi n_o \sigma^2 r_o} \, . \qquad \ldots \quad (8)$$

Jones and Milne used eqn. (8) to calculate the levels of escape of hydrogen and helium, which at the time they wrote the papers were believed to be the constituents of the upper atmosphere. Assuming the temperature of the outer atmosphere to be 219°K they obtained the values of the levels of escape of the above two gases to be 1521 km. and 630 km. respectively (heights measured from above the stratosphere, 20 km. from the surface of the earth). We now know that the assumptions of Milne and Jones regarding the constituents as well as the temperature of the upper atmosphere are wrong. According to modern concep-

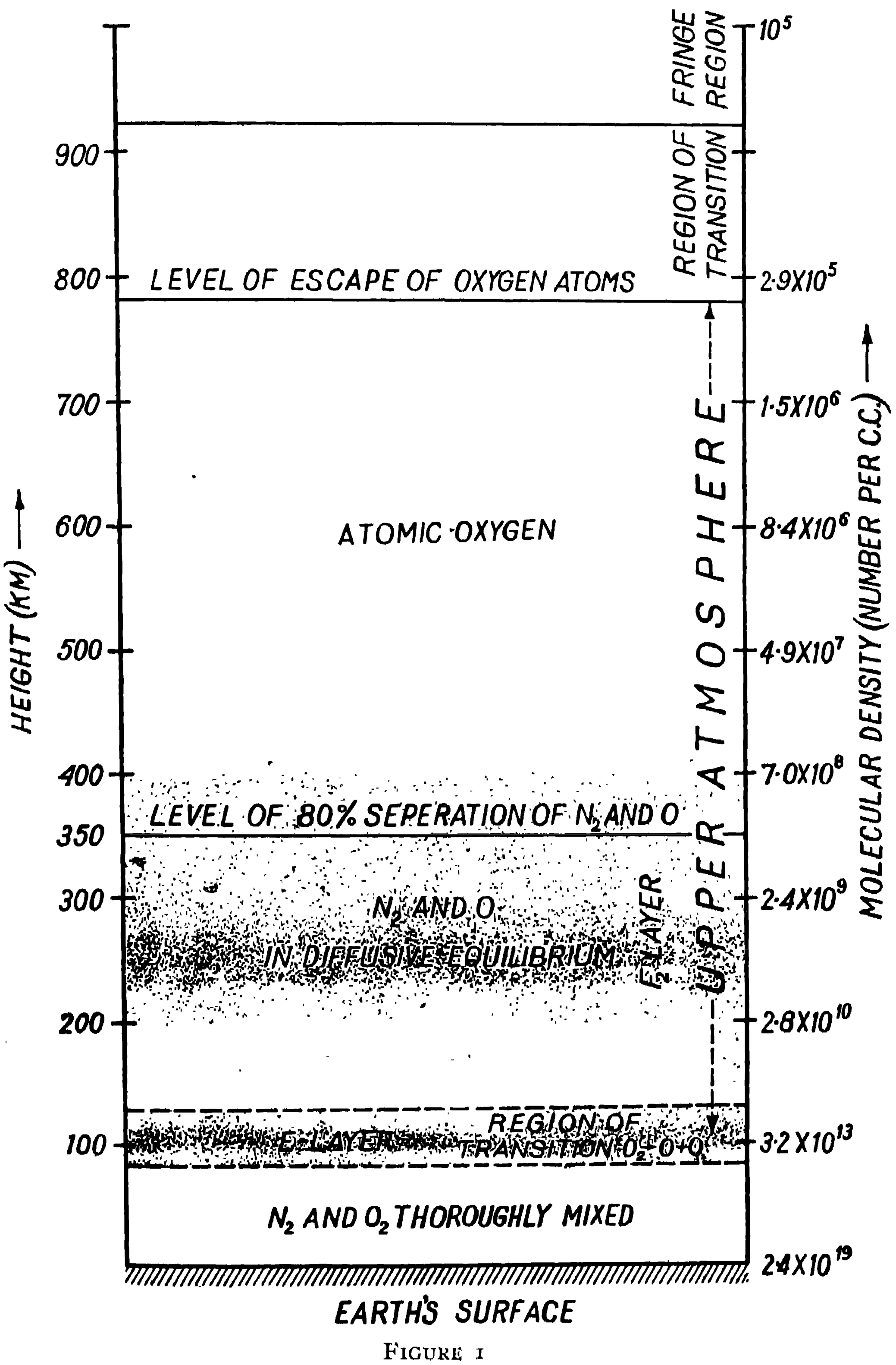
FRINGE REGION
10^5
REGION OF TRANSITION
900
800
LEVEL OF ESCAPE OF OXYGEN ATOMS
2·9X10^5
700
1·5X10^6
600
ATOMIC OXYGEN
8·4X10^6
500
4·9X10^7
400
7·0X10^8
LEVEL OF 80% SEPERATION OF N2 AND O
350
300
N2 AND O
IN DIFFUSIVE EQUILIBRIUM
F2 LAYER
2·4X10^9
UPPER ATMOSPHERE
200
2·8X10^10
REGION OF TRANSITION O2→O+O
E LAYER
100
3·2X10^13
N2 AND O2 THOROUGHLY MIXED
2·4X10^19
EARTH'S SURFACE
HEIGHT (KM)
MOLECULAR DENSITY (NUMBER PER C.C.)

FIGURE 1

tions, the atmosphere above 100 km. consists of molecular nitrogen and atomic oxygen and is at a high temperature.[7] Further, explorations by radio waves as well as observations on the meteorological phenomena afford us means of estimating the molecular density at 100 km level. We can, therefore, calculate the minimum heights of the levels of escape of the above two gases by taking 100 km. level as the datum level and also assuming the most probable temperature. Taking[8] $(n_o)_O = 6{\cdot}963 \times 10^{12}$/c.c., $(n_o)_{N_2} = 2{\cdot}543 \times 10^{13}$ /c.c., $\sigma_o = 2{\cdot}63 \times 10^{-8}$ cm., $\sigma_{N_2} = 3{\cdot}8 \times 10^{-8}$ cm., $T = 1000°K$, we find the level of escape of N_2 and of O to be 410 km. and 671 km. respectively (heights measured from above 100 km. level). These values, as mentioned before, give a lower estimate of the height of the level of escape. To estimate the upper limit of this level, from above which molecules may emerge to form the spray, we make θ approach 90°. Putting $\theta = 85°$ the levels for N_2 and O are found to be 485 km. and 831 km. (above the 100 km. level). It is to be noted that all the molecules above this level which avoid collisions will not escape from the atmosphere. Only those molecules which move with velocities greater than $\frac{2ga^2}{r}$ will get rid of the earth's attraction. This critical velocity for escape of a molecule from the top of the earth's atmosphere is calculated to be about 11 km./sec. ($r = 7378$ km., $a = 6378$ km., $g = 732$ cm./sec.^{2} at 1000 km. level). The distribution of the constituent gases and molecular density at different altitudes of the atmosphere are shown in Figure 1.

§4. EFFECT OF IONIZATION ON THE RATE OF ESCAPE OF MOLECULES FROM THE EARTH'S ATMOSPHERE

In the previous section we have seen that a neutral particle requires a velocity of about 11 km./sec. to overcome the pull of gravity and escape from the earth's atmosphere. We have not considered, however, the possibility of the atoms and molecules in the fringe region being ionized by the ultra-violet radiation of the sun. The motion and distribution of the particles, if ionized, will be profoundly influenced by the earth's magnetic field. The critical velocity for escape of these ionized particles and its variation with latitude may be estimated after the elaborate calculations of Störmer[9] on the trajectories of charged particles coming from infinity towards a magnetic dipole. His calculations show that there is a space Q_γ characterized by a certain value of γ, an integration constant, within which the charged particles cannot enter. For $\gamma < -1$, no charged corpuscle can reach the magnetic dipole. For γ lying between -1 and 0, charged particles from infinity may reach the magnetic dipole. There is, however, a toroidal space round the dipole where they cannot enter. The meridional curve of this space is given by—

$$r = \frac{\sqrt{\gamma_1^2 + \sin^2\chi} - \gamma_1}{\sin\chi} \qquad \dots \quad (9)$$

where $\gamma_1 = -\gamma$;

χ = the angle which the radius vector r makes with the magnetic axis z ;

r = the radius vector measured in units of length $C = \sqrt{\frac{M}{HR}}$, M being the magnetic moment of the dipole ;

and $HR = \frac{m}{e} v$, where v is the velocity of the charged particle.

The maximum angular distance χ_1 of the zone within which the charged particles may enter the earth's atmosphere (the earth is regarded as a small sphere of radius a placed round the magnetic dipole) is obtained by finding the intersection of the toroidal surface with a sphere of radius a. This is shown in Figure 2. It can be shown from eqn. (9) that χ_1 is given by

$$\sin \chi_1 = \sqrt{2\gamma_1 a}\ , \quad \left| \begin{array}{l} a \text{ is measured} \\ \text{in unit of length} \quad C = \sqrt{\frac{M}{HR}} \end{array} \right] \quad \begin{array}{l} \text{or since the maximum} \\ \text{value of } \gamma_1 \text{ is } 1, \end{array}$$

$$\text{Sin } \chi_1 = \sqrt{\frac{2a}{C}}\ , \quad \Big[\ a \text{ measured in cms.} \qquad \ldots \quad (10)$$

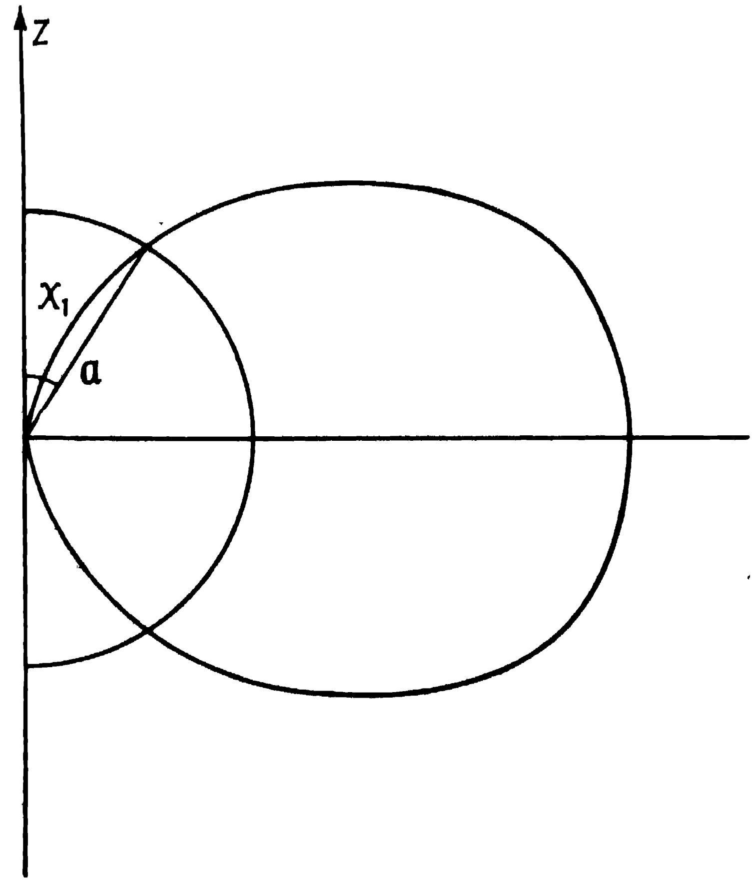

FIGURE 2

The case of charged particles escaping from the earth's atmosphere being exactly complementary to that of Störmer's, we can calculate the value of χ_1 giving the zone from which a charged particle having a given velocity v can escape from the earth's atmosphere to infinity. Table I is prepared following the above method, and shows that the velocity required by an ion of atomic oxygen to disentangle itself from the earth's magnetic field is much greater than that required for a neutral atom. Whereas the required velocity for a neutral particle is independent of the latitude, that for a charged particle is not so. The critical velocity increases with decreasing latitude.

TABLE I

Co-latitude χ_1 (in degrees)	$HR = \frac{m}{e} \cdot v.$	$C = \sqrt{\frac{M}{HR}}$ (cm.).	Velocity of escape (cm./sec.).
5°·5	$3\cdot344 \times 10^3$	$1\cdot585 \times 10^{11}$	2×10^6
8°·3	$1\cdot672 \times 10^4$	$7\cdot087 \times 10^{10}$	10^7
14°·8	$1\cdot672 \times 10^5$	$2\cdot242 \times 10^{10}$	10^8
27°·2	$1\cdot672 \times 10^6$	$7\cdot087 \times 10^9$	10^9
54°·35	$1\cdot672 \times 10^7$	$2\cdot242 \times 10^9$	10^{10}
90°	$4\cdot18 \times 10^7$	$1\cdot418 \times 10^9$	$2\cdot5 \times 10^{10}$

In calculating the rate of escape of molecules, particularly of light gases like hydrogen or helium, no account has hitherto been taken of the possibility of ionization of the atoms in the fringe region and their subsequent entanglement in the magnetic field. If this is taken into consideration, the rate of escape will be greatly diminished. The rate will also be different for different latitudes. To make an estimate of the actual rate of escape taking this process of ionization into account we require a precise knowledge of the "ionization probability" of the gaseous constituents of the atmosphere, the possible sources of particles possessing the requisite speed and their distribution at different levels. Lack of necessary data prevents us from attempting such quantitative calculation.

§5 ROTATION OF THE EARTH—VISCOUS DRAG

An interesting point that is to be considered is the possibility of different layers of the atmosphere at different heights moving with different angular velocities on account of insufficient viscous drag. The expression for the coefficient of viscosity of a gas being $\eta = \frac{1}{3} N m \bar{c} \lambda$, it would appear that the viscosity is independent of pressure. The above expression will, of course, hold as long as the gas obeys gas kinetic laws,[9] *i.e.*, as long as collisional frequency is appreciable. It may be noted here that in experimental investigations on the viscosity of a gas at low pressure, with laboratory apparatus, the coefficient of viscosity falls to a low value at low pressure. It may be shown, however,[10] that this is due to the size of the vessel becoming comparable with the mean free path of the gas molecules. As long as the size of the apparatus is large compared with the mean free path (as is the case for the atmosphere covering the earth), the coefficient of viscosity should remain independent of pressure. The effective drag of one layer of the gas upon the other depends not on the coefficient of viscosity alone but on the so-called coefficient of kinematical viscosity [11,12] which is defined as $\frac{\eta}{\rho}$. It would then appear that the drag of one layer upon another is actually greater in the high atmosphere than in the lower atmosphere. We give a curve (Figure 3) showing the relation between the change of kinematical viscosity with height from 100 km. level for the atmospheric distribution assumed above.

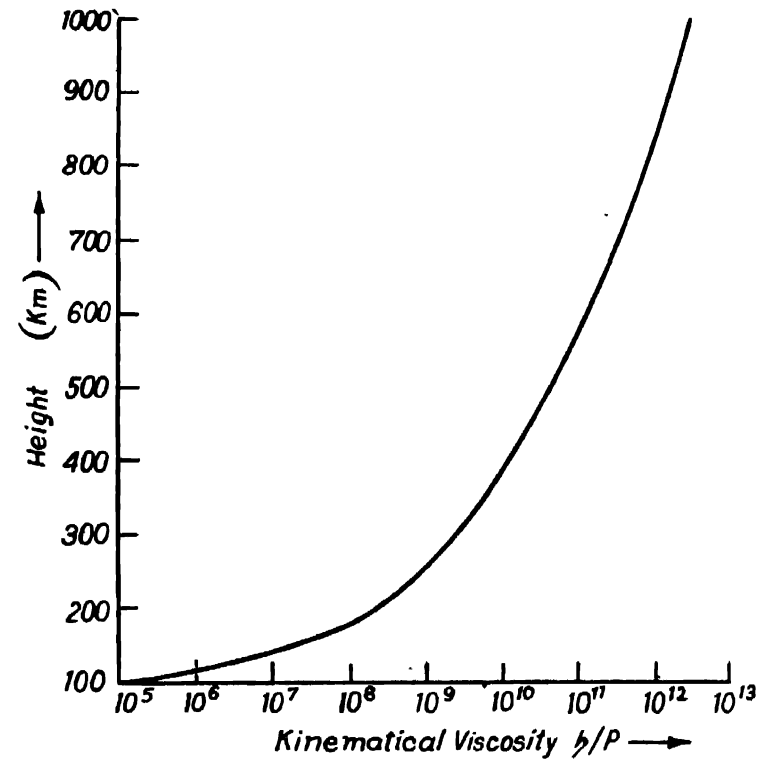

FIGURE 3

We are, therefore, justified in assuming that up to the limit of the outer atmosphere within which collisions are appreciable the air molecules participate in the rotation of the earth. A molecule leaving the lower region from the level of escape and moving in the fringe region with the velocity acquired at the last collision, will, therefore, have a component of velocity in the direction of rotation of the earth. As a consequence of this the trajectory of such a molecule will experience a curvature opposite to the direction of rotation of the earth, *i.e.*, the longitude at which the molecule will return to the atmosphere from the fringe region will be different from that at which it left the level of escape.

§6. FRINGE REGION OF THE ATMOSPHERE

We have already referred in §2 to the fringe or the spray region of the atmosphere where molecules move freely with the velocity acquired at the last collision in the lower region. Being subject only to the pull of gravity they describe parabolic, elliptic or hyperbolic paths according to the magnitude and direction of their velocity. It is evident from what has been said in §3 that the spray region begins from above the critical level. It is possible to calculate the average number of particles per c.c. between any two heights in the spray region from a knowledge of the density and the temperature round the level of escape. We proceed to carry out the calculation on the assumption that the level of escape begins from about 800 km., and that the molecular density and temperature here are $3{\cdot}0 \times 10^5$/c.c. and 1000°K. The atmospheric constituent, here, is practically wholly atomic oxygen.

TABLE II

Region (Z_2-Z_1) in km. above earth's surface.	Velocity (km./sec.).		Number crossing/cm.^{2} of 1000 km. level.		Average density (Molecules per c.c.).
	v_2	v_1	n_2	n_1	
1250-1000	1·87	1·5	$7{\cdot}8 \times 10^{12}$	10^{13}	$1{\cdot}0 \times 10^5$
1500-1250	2·6	1·87	4×10^9	$7{\cdot}8 \times 10^9$	$3{\cdot}0 \times 10^4$
1750-1500	3·15	2·6	$2{\cdot}4 \times 10^8$	4×10^9	$8{\cdot}0 \times 10^3$
2000-1750	3·59	3·15	$1{\cdot}38 \times 10^7$	$2{\cdot}4 \times 10^8$	$4{\cdot}5 \times 10^2$
2250-2000	3·95	3·59	$1{\cdot}07 \times 10^6$	$1{\cdot}38 \times 10^7$	$2{\cdot}3 \times 10$
2500-2250	4·27	3·95	$9{\cdot}2 \times 10^4$	$1{\cdot}07 \times 10^6$	2·0
2750-2500	4·54	4·27	$9{\cdot}9 \times 10^2$	$9{\cdot}2 \times 10^4$	$1{\cdot}0 \times 10^{-1}$
3000-2750	...	...	...	...	...

Assuming a Maxwellian distribution in the region of the level of escape and also taking the variation of '*g*' with height into account Table II is prepared, giving the average molecular density at different levels of the region. The source of particles in the fringe region is assumed to be the molecules between 800-1000 km. level. The second column gives the range of velocity required to send the molecules flying within the region cancerned ; v_1 is the initial velocity required to send the particle above the lower level of the zone and v_2 above the upper level. The third column gives the number of particles per square cm. crossing the 1000 km. level, n_2 corresponding to those that reach the upper level and n_1 to the lower. The last column gives the average molecular density at different altitudes of the spray or fringe region beginning from 800 km.

EXTENSION OF THE FRINGE REGION BY SUPER-ELASTIC COLLISIONS

From Table II it will be seen that the molecular density in the spray region rapidly diminishes with increasing height and at 2500 km. from the earth's surface, the density falls to 1 molecule per c.c. which is the order of density in the inter-stellar space. According to Hulburt,[2] however, the spray region may extend to heights much beyond this, if it be assumed that a certain fraction of the molecules in the region of the level of escape acquire high velocity due to super-elastic collisions. There are, according to Hulburt, 10^{16} molecules in a vertical column per sq. cm. in the fringe region (according to Hulburt the fringe region begins from 450 km. but as we have seen in §3 this should be from 770 km.). These 10^{16} molecules suffer 10^{11} collisions per sec. A fraction (10^{-8}) of these is of the super-elastic kind which impart velocities of the order of 10 km./sec. to the neutral particles. Such particles rise to heights of 40,000-50,000 km. above the centre of the earth, in about 3 hours time. Hulburt utilizes these high-velocity particles for explaining the upper atmosphere phenomena like aurorae and magnetic storms. These he suggests are due to the high-flying particles getting ionized by the solar ultra-violet rays and reaching the higher latitudes by being entangled by the earth's magnetic field. We will in the following sections discuss in some detail the above-mentioned assumptions of Hulburt on which this attractive theory of magnetic storms and aurorae is based.

We will discuss the matter in two steps :—

1. Firstly, we will discuss the spectroscopic processes which in the region round the level of escape may lead to the production of high-velocity particles by super-elastic collisions and calculate the number of the high velocity particles produced per second thereby, and

2. Secondly, the motion of these particles, when ionized, in the magnetic field of the earth.

POSSIBLE SOURCES OF SUPER-ELASTIC COLLISIONS

In the super-elastic type of collision, the encounter takes place between a neutral and an excited particle. The excited particle might give up whole or part of its energy to the neutral particle and the latter may thus acquire speed corresponding to the energy lost by the excited particle. In order to estimate the number and the speed of the high-velocity particles it is necessary first to study the nature of the excited particles. Now an ordinary excited atom or molecule has a spontaneous chance of coming down to a lower state of energy with emission of radiation. This chance is very high—in other words, the life of an atom in an excited state is very small. In fact both theoretical considerations and experimental results show that the life of an ordinarily excited atom is of the order of 10^{-9} to 10^{-10} sec. It is thus obvious that in order that the particle may lose its energy by collision instead of by spontaneous radiation, the average number of collisions per sec. must be of the same order as the inverse of the life of the excited atom. Now in the region of the atmosphere we are considering, the collisional frequency is much smaller than the inverse of the average life of ordinary excited atoms or molecules, that is, several orders less than 10^8 or 10^9/sec. The excited particles which may possibly suffer super-elastic collisions cannot, therefore, be of the ordinary excited types, but must be of the metastable type for which the life is very much longer. We will thus have to enquire about the possible metastable states to which the atoms and molecules present in the outer atmosphere may be excited and which by superelastic collisions could produce high-speed neutral particles. The particles in the upper atmosphere which it is necessary to consider are N_2, N and O. It is now definitely established [13] that molecular oxygen, as such, is not present in the high levels of the atmosphere owing to the dissociative action of the solar ultra-violet rays. Recent spectroscopic studies of the light of the night sky seem to show that N_2 is also dissociated, producing atomic nitrogen.[14,15] The extent to which this dissociation is effected and the relative proportions of N_2 and N have not yet been quantitatively worked out. We will consider below, in some detail, the metastable states of the these three gases.

ATOMIC OXYGEN

The presence of metastable oxygen atoms in the upper atmosphere in the ^1S and ^1D states is well established by spectroscopic evidence. The famous green line $\lambda 5577$ and the two red auroral lines $\lambda 6300$ and $\lambda 6363$ are due to the transitions $^1S-{}^1D$ and $^1D-{}^3P_{1,2}$. The life of oxygen atom in these states (^1S and ^1D) has been found by wave-mechanical calculations by Condon[16] to be $\cdot 5$ and $1\cdot 3 \times 10^2$ secs. respectively.

It may be interesting to discuss how oxygen atoms are brought to these metastable states. Chance of such transitions occurring by direct absorption is extremely small. It is sometimes suggested that though direct absorption may be inoperative, the atom may be brought to the metastable state through a roundabout process. According to A. K. Das,[17] the atom may be raised to a higher energy level by a direct absorption of radiation and then come down to a metastable state. A study of the energy-level diagram of oxygen atom shows, however, that there is no such higher energy level to which the atom may be raised from the ground-state and from which it may drop directly to a metastable state. It is possible, of course, to reach these metastable states by several steps, but obviously the number of atoms reaching the metastable states by this process will be extremely small. It seems that the most likely process by which the atom may be excited to metastable states are photo-dissociation and/or recombination of oxygen ions with electrons according to the following scheme :—

$$(1)\quad (a)\quad O_2 + h\nu_0 \rightleftharpoons O\,{}^3P + O\,{}^1D \qquad (11)$$

$$\lambda_0 = 1750\text{Å}$$

$$(b)\quad O_2 + h\nu_0 \rightleftharpoons O\,{}^3P + O\,{}^1S \qquad (12)$$

$$\lambda_0 = 1325\text{Å}$$

$$(2)\quad O^+ + e \rightleftharpoons O\,{}^3P \text{ or } O\,{}^1D \text{ or } O\,{}^1S \quad \ldots \quad (13)$$

There is experimental evidence [18] that the first of the two absorption processes actually occurs. The second absorption process, though quite likely, has not yet been observed experimentally.

With regard to the recombination process it would appear that owing to the presence of ions in the outer region it is to be the most likely process of producing metastable oxygen atoms.

MOLECULAR NITROGEN

The existence of nitrogen molecules in the metastable state $A\,{}^3\Sigma_u^+$ is evinced by the Vegard-Kaplan band systems observed in the light of the night sky. The transitions corresponding to these bands are given by $A\,{}^3\Sigma_u^+ \rightarrow X\,{}^1\Sigma$ and are ordinarily forbidden.

The process by which N_2 molecules attain the $A^3\Sigma_g^+$ state is as follows: By direct absorption of solar ultra-violet radiation N_2 molecules are raised to higher energy-levels (for instance $X^1\Sigma \rightarrow a^1\pi_u$, $X^1\Sigma \longrightarrow b$, $X^1\Sigma \longrightarrow b$, $X^1\Sigma \longrightarrow c$, $X^1\Sigma \longrightarrow f$, $X^1\Sigma \longrightarrow g$, $X^1\Sigma \longrightarrow h$) and are finally ionized ($X^1\Sigma \longrightarrow A^1$).

N_2 ions combine with electrons to form D, $C^3\pi_u$, $B\pi_g$. These by spontaneous transition and emission of IV^{+ve}, II^{+ve} and I^{+ve} bands drop to $A^3\Sigma_g^+$ level. Being highly metastable it can give up its energy by collision or by spontaneous radiation in a region where the interval between collisions is much larger than the average life.

Active nitrogen[19] which is sometimes suggested as the source of metastable oxygen atoms may also produce high-speed particles by super-elastic collisions. Our knowledge of active nitrogen both as regards its nature and origin and also its possible presence in the upper atmosphere is extremely meagre. It is, therefore not possible to consider in detail the possible effect of active nitrogen on the production of high-speed particles.

ATOMIC NITROGEN

The following photo-dissociation processes may convert nitrogen molecules to nitrogen atoms :—

$$N_2(^1\Sigma_g) + h\nu_0 \rightleftharpoons N^4S + N^2D$$

$$\lambda = 1273\text{Å}$$

$$N_2(^1\Sigma_g) + h\nu_0 \rightleftharpoons N^4S + N^2P$$

$$\lambda_0 = 1124\text{Å}.$$

No such continuous absorption corresponding to these processes has yet been observed in the laboratory for N_2. Only very recently spectroscopic evidence regarding the presence of atomic nitrogen in the metastable state has been adduced by the discovery of lines corresponding to the transitions $^2D \longrightarrow {}^4S$, and $^2P \longrightarrow {}^4S$ in the night sky spectrum. The third forbidden line of atomic nitrogen lines in is the infra-red and has not yet been investigated.

Nitrogen atoms in metastable states might be obtained by the above photo-dissociation processes or by the predissociation of nitrogen molecules in highly excited vibrational states, as has been suggested by Saha.[20]

In Table III are given the velocities and heights which may be attained by oxygen atoms and nitrogen molecules respectively if they suffer super-elastic collisions with O, N, and N_2 molecules in various metastable states.

TABLE III

Source.		Energy (in electron-volts).	Velocity (in km./sec.) imparted to		Height (in km.) attained (above earth's centre).	
			O	N_2	O	N_2
O	$^1D-{}^3P$	2	4·89	3·7	9,500	8,500
	$^1S-{}^3P$	4·2	7·09	5·36	14,000	10,000
N	$^2P-{}^2D$	1·19	3·77	2·85	8,500	8,000
	$^2D-{}^4S$	2·37	5·96	4·03	10,000	8,700
	$^2P-{}^4S$	3·56	6·53	4·93	12,000	9,500
N_2	$N_2\ A^3\Sigma_u \rightarrow X^1\Sigma_g$	6·15	8·58	6·49	23,000	12,000
	Active $N_2 \rightarrow X^1\Sigma_g$	9·71	10·79	8·13	97,000	19,000

Of the various metastable states shown in Table III the highest energies for super-elastic collision are given by the metastable $A^3\Sigma_u^+$ state of N_2 and also by the active nitrogen. But since N_2 is confined more or less below the region of escape owing to diffusive separation, particles suffering super-elastic collisions with such N_2 molecules will not be able to attain great heights. The metastable states of atomic oxygen seem, therefore, to be the most probable sources of the high-speed particles of Hulburt. Atomic nitrogen if it exists can be expected to reach the highest levels on account of its being lighter than atomic oxygen and will undoubtedly lie above the level of escape. But it will be seen from Table III that the energy of its metastable states is on the whole smaller than the $^1S-{}^3P$ energy of atomic oxygen. In order, therefore, to estimate the number of particles shot up by super-elastic collisions, it will suffice to calculate the number of oxygen atoms in the ^1S state. This we proceed to do below.

CALCULATION OF THE NUMBER OF METASTABLE OXYGEN ATOMS

Since the most likely process by which oxygen atoms may be excited to the ^1S state is that of recombination of negative oxygen ions with electrons as shown in equation (13), we will have to calculate, in the first instance, the number of O-ions produced per sec. in a column of 1 sq. cm. cross-section above 770 km., *i.e.*, above the level of escape.

The number of ions produced per second in a column of 1 sq. cm. cross-section above 770 km. level may be calculated, making the simplifying assumption that the ionizing radiation is very little diminished in intensity by the slight absorption it suffers in its passage through the spray region. (This is consistent with the picture of the spray region given before, namely, that above the critical level there is very little screening of one atom by another.) Thus the number of ions produced per second, in the column, is given by

$$q = \mathrm{N}\int_{\nu_0}^{\infty} \mathrm{I}(\nu)_0 \psi(\nu) d\nu \qquad \dots \quad (14)$$

where N = total number of atoms in the column ;

$\mathrm{I}(\nu)_0$ = intensity of the ionising radiation outside the earth's atmosphere ;

$\psi(\nu)$ = ionization probability $= \dfrac{\tau(\nu)}{h\nu}$ where $\tau(\nu)$ = absorption per atom.

Assuming the sun to be radiating like a black body at a temperature T_s,

$$\mathrm{I}(\nu)_0 = \frac{8\pi\beta h\nu^3}{c^2} e^{-\frac{h\nu}{k\mathrm{T}_s}}$$

where $4\pi\beta$ is the solid angle subtended by the sun at the earth. Thus

$$q = \mathrm{N}.\frac{8\pi\beta}{c^2}\int_{\nu_0}^{\infty} \nu^2 e^{-\frac{h\nu}{k\mathrm{T}_s}} \tau(\nu) d\nu$$

which comes to

$$q = \mathrm{N}.\frac{8\pi\beta.\tau_0 \nu_0^2 k\mathrm{T}_s}{c^2 h} e^{-\frac{h\nu_0}{k\mathrm{T}_s}}$$

where τ_0 corresponds to the threshold frequency ν_0.

Taking the value for τ_0 as given by Saha and Rai [21] ($2{\cdot}81 \times 10^{-17}$) we get the number of oxygen ions formed per second in the column to be given by

$$q = 5{\cdot}667 \times 10^8.$$

If Chapman's [22] value for $\tau_0 (3{\cdot}5 \times 10^{-16})$ is taken, then

$$q = 5{\cdot}043 \times 10^9.$$

Thus we find that about $5{\cdot}7 \times 10^8$ to 5×10^9 atoms out of the total 10^{16} atoms are ionized per second.

Next we will find how many of these ionized atoms recombine per second to form neutral atoms in the excited ^1S state. This number is given by

$$q = q_+ q_- \alpha \qquad \dots \quad (15)$$

where q_+ = number of positive ions ;

q_- = number of electrons ;

α = recombination coefficient for capture in the excited ^1S state.

In the absence of any experimental value for the coefficient of recombination, α, we have to fall back upon theoretical value. The recombination coefficient is given by

$$\alpha = q(n, l) \sqrt{\frac{2eV}{m}} \quad \dots \quad (16)$$

where $e.V$ = the energy of the recombining electron and $q(n, l)$ is cross-the section for electron capture in the particular orbit and is given by

$$q(n, l) = \frac{A(n, l) \times 10^{-20} . Z^2}{V} \quad \dots \quad (17)$$

Z is the effective atomic number and $A(n, l)$ a factor depending on the orbit in which the electron is captured.

We take the value of $A(n, l)$ for the ^1S state from the theoretical curves given by Morse and Stueckelberg.[23] Thus for a temperature of 1000° K, V = ·13 *e.v.* and we have, taking $Z = 3$,

$$q(n, l) = \frac{\cdot 035 \times 10^{-20}}{\cdot 13} . 3^2$$

and consequently

$$\alpha = 4{\cdot}969 \times 10^{-13}.$$

Thus the number of ions recombining per sec. in the ^1S state is given by

$$\begin{aligned} q &= [5{\cdot}7 \times 10^8]^2 \times 4{\cdot}969 \times 10^{-13} \\ &= 1{\cdot}6 \times 10^5, \text{ taking } q = 5{\cdot}7 \times 10^8 \\ \text{or} \quad &= [5{\cdot}04 \times 10^9]^2 \times 4{\cdot}696 \times 10^{-13} \\ &= 1{\cdot}3 \times 10^7, \text{ taking } q = 5{\cdot}04 \times 10^9. \end{aligned}$$

AVERAGE MOLECULAR DENSITY IN THE FRINGE REGION

We next consider how many of these 10^5 to 10^7 atoms suffer super-elastic collisions per sec. The mean free-path of an atom round the 800 km. level or so is on the average about 150 km. The thermal velocity corresponding to 1000°K is about 2 km./sec. So that in the region between 770 km. and 930 km. an atom suffers one encounter in about 75 secs. The life of the atom in the ^1S state being ·5 sec., one atom only out of 150 suffers an encounter per sec. On an average, therefore, 10^3 to 10^5 atoms per sec. receive high velocity due to super-elastic collisons. Roughly we can take that about 10^4 atoms are shot up per sec. per sq. cm. in the spray region. Considering now the oxygen atoms in ^1D state we find that only 10^3 to 10^5 atoms out of the 10^5 to 10^7 atoms in the ^1S state suffer super-elastic collisions per sec. in the column above 770 km. level. Thus most of the atoms in the ^1S state come down to the ^1D state by emitting the green line. The life in the ^1D state being much longer (10^2 secs.) these atoms come down to the ground state by super-elastic collisions. Thus it may be taken that roughly 10^6 atoms with 4·89 km./sec. velocity ($^1D-^3P$) and 10^4 atoms with 7·09 km./sec. velocity are shot out per sec. through each sq. cm. area of 1000 km. level. We have to see, therefore, how the density distribution in the spray region is modified by these high-flying atoms reaching 8500 and 14000 km. respectively. Table IV is prepared after the method followed in preparing Table II and depicts how the super-elastic collisions affect the merging of the atmosphere with the inter-steller space. This occurs at a distance somewhat greater from the centre of the earth than when the effects of super-elastic collisions are not considered.

TABLE IV

Region (Z_2-Z_1) in kms above earth's surface.	Number crossing per cm.^{2} of datum level at Z = 1000 km.	Average density (Molecules per c.c.)
1750-1500	4×10^9	$8·0 \times 10^3$
2000-1750	$2·4 \times 10^8$	$4·5 \times 10^2$
2250-2000	$1·48 \times 10^7$	25
2500-2250	$2·07 \times 10^6$	5
3000-2500	1×10^6	3
4000-3000	10^4	10^{-1}

PART II. THE ULTRA-VIOLET THEORY OF AURORAE AND MAGNETIC STORMS

§1. INTRODUCTION

In Part I we have calculated the number of atoms which are shot up across the level of escape and which ultimately contribute to the formation of the spray region. We have not considered the effect of ionization of these particles which would undoubtedly occur by the absorption of the ultra-violet radiation of the sun. As mentioned in the introduction the ionized particles will in general proceed polewards and might contribute to the production of aurorae and magnetic storms.[2] We now proceed to consider these phenomena in some detail.

We have seen that 10^6 oxygen atoms with velocity of 5·0 km./sec. and 10^4 atoms with velocity of 7·0 km./sec will be shot across every sq. cm. surface from the region of escape into the outer atmosphere. In the absence of any other physical process, these high-speed atoms will attain heights of 8500 km. and 14000 km. respectively from the centre of the earth. The super-elastic collisions will thus extend the spray region to about 14000 km.

We will now consider the effect of ionizing solar radiation. Such ionization by the incident solar ultra-violet radiation will profoundly modify the motion and distribution of the high-flying particles. The magnetic field of the earth will cause the ionized particles to spiral round the magnetic lines of force and lead them towards the polar regions of the earth. We discuss first in some detail the path of a charged particle when it is subject simultaneously to the influence of the terrestrial magnetic field and also to the pull of gravity.

§2. MOTION OF IONS IN GRAVITATIONAL-MAGNETIC FIELD

Let us consider the simple case of an ion moving in a constant magnetic field H with velocity v_0. If the velocity v_0 be in a direction making an angle θ with the magnetic line of force, it will have components $v_0 \cos\theta$ and $v_0 \sin\theta$ along and at right angles to the magnetic line of force. The component along the magnetic line of force will remain unaffected by the magnetic field but the component at right angles to the field will make the ion spiral round the magnetic line of force. Let the component at right angles to the magnetic field be denoted by v; then the equation of motion of the ion is

$$m\frac{dv}{dt} = \frac{e[v \times \mathrm{H}]}{c}. \qquad \dots \quad (18)$$

The motion described by (18) is the motion with constant velocity v in a circle of radius R given by

$$\mathrm{R} = \frac{mc}{\mathrm{H}e} \cdot v. \qquad \dots \quad (19)$$

The angular velocity in the circular path is

$$\omega = -\frac{e\mathrm{H}}{mc}. \qquad \dots \quad (20)$$

Thus component $v_0 \sin\theta\ (=v)$ of the ion makes it move in a helical path of radius $\mathrm{R}\left(=\frac{mc}{\mathrm{H}e} \cdot v\right)$ while the component $v_0 \cos\theta$ simply leads the ion along the magnetic line of force.

Let us now consider the additional effect due to the presence of the gravitational field on the ion paths.[24] The force equation in this case becomes

$$m\frac{dv}{dt} = \mathrm{F} + \frac{e[v \times \mathrm{H}]}{c} \qquad \dots \quad (21)$$

where $\mathrm{F} = mg$ is the force due to pull of gravity.

To transform eqn. (21) in the form of eqn. (18) we suppose a velocity $u_g = \frac{c[F \times H]}{cH^2} = c. \frac{mg}{eH}$ to be imparted to the ion. The velocity of the ion is now given by $V = (v - u_g)$ and eqn. (21) may be written

$$m \frac{dV}{dt} = e[V \times H]$$

and consequently

$$R = \frac{mc}{He} \cdot V$$

$$= \frac{mc}{He} (v^2 - 2\vec{u}_g \vec{v} + u_g^2)^{\frac{1}{2}} .$$

Thus the effect of the force $F = mg$ is firstly to change the radius of the circular path R from $\frac{mc}{He} \cdot v$ to $\frac{mc}{He} (v^2 - 2\vec{u}_g \vec{v} + u_g^2)^{\frac{1}{2}}$ and secondly to make the helical path advance in a direction at right angles both to g and to H with a velocity $u_g = \frac{mg}{He} \cdot c$.

It is easy to see, therefore, that the atoms as soon as they are ionized in the spray region will, owing to the component of velocity at right angles to the magnetic field, begin to trace cycloidal paths, *i.e.*, they will spiral round the magnetic lines of force and have at the same time a drift towards the east for positive ions and towards the west for electrons and negative ions. The ions and electrons will be led at the same time by the component of velocity along the line of force towards the north pole or south pole according to the direction of motion at the time of ionization.

§3. EFFECT OF VARIATION OF g AND H ON ION PATHS

In calculating the helical paths of ions and electrons in the fringe region, we have not taken into account the variation of g and H with altitude. These variations will affect three things : (*a*) the radius of curvature of the cycloidal path of the ion, R $\left(= \frac{mc}{He} \cdot V \right)$, (*b*) the angular velocity of rotation in the cycloidal path $\omega \left(= - \frac{cH}{mc} \right)$ and (*c*) the gravitational-magnetic drift velocity $u_g \left(= c. \frac{mg}{He} \right)$.

(*a*) As the ion descends downwards H increases continually inversely as the cube of the distance of the point under consideration from the centre of the earth. The radius of curvature of the ion path thus decreases continually downwards.

(*b*) As H increases downwards, the angular velocity of rotation will increase in the same rate as that of (*a*).

(*c*) Since g varies inversely as the square and H inversely as the cube of distance from the centre of the earth and since these are in the numerator and denominator respectively in the expression for the drift velocity, the drift velocity will decrease owing to the relatively greater increase of H.

Simply stated the above results mean that as the ion comes downwards, the pitch of the helical path becomes smaller and smaller, the rotation in the orbit becomes faster and faster and the helical path tends more and more to be confined to one particular line of force. We give below a Table showing the variation of R, ω, $u_{//}$ for a typical case. [$V = 1$ km./sec., $m = 16 \times 1{\cdot}662 \times 10^{-24}$ gm.]

TABLE V

Height above earth surface Z (in km.).	g (cm./sec.^{2}).	H (Gauss)	R (cm.).	ω (radian/sec.).	$u_{//}$ (cm./sec.).
1,000	732·6	·37	$4{\cdot}48 \times 10^2$	$2{\cdot}23 \times 10^2$	3·28
2,000	568	·253	$6{\cdot}56 \times 10^2$	$1{\cdot}52 \times 10^2$	3·73
3,000	452	·174	$9{\cdot}52 \times 10^2$	$1{\cdot}05 \times 10^2$	4·31
4,000	370	·128	$1{\cdot}3 \times 10^3$	$7{\cdot}7 \times 10^1$	4·81
5,000	308	·095	$1{\cdot}74 \times 10^3$	$5{\cdot}74 \times 10^1$	5·36
10,000	148·6	·0286	$5{\cdot}8 \times 10^3$	$1{\cdot}72 \times 10^1$	8·64
20,000	57·3	·0054	$3{\cdot}1 \times 10^4$	3·23	17·74
40,000	18·6	·0007	$2{\cdot}29 \times 10^5$	·43	39·06

Figures 4, 5 and 6 are drawn showing the variation of R, ω and $u_{//}$ with height as the ion comes downwards.

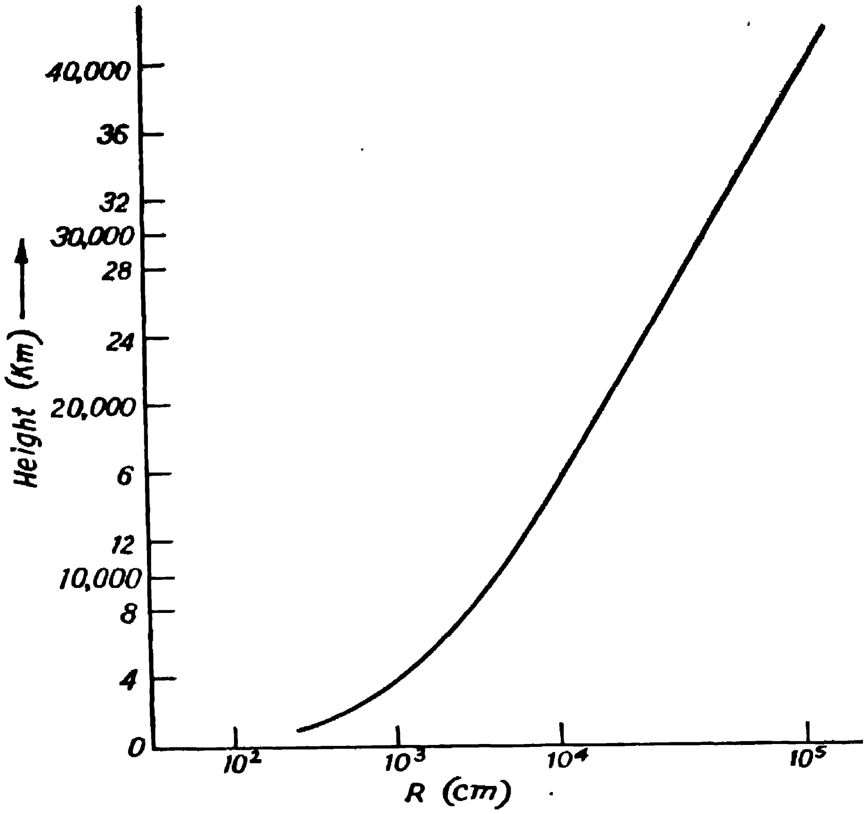

FIGURE 4

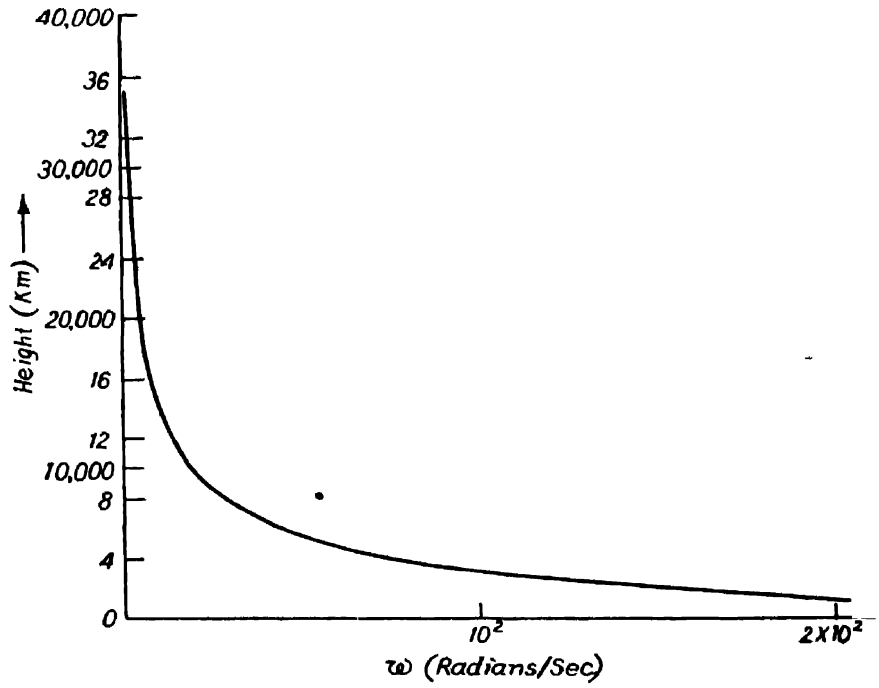

FIGURE 5

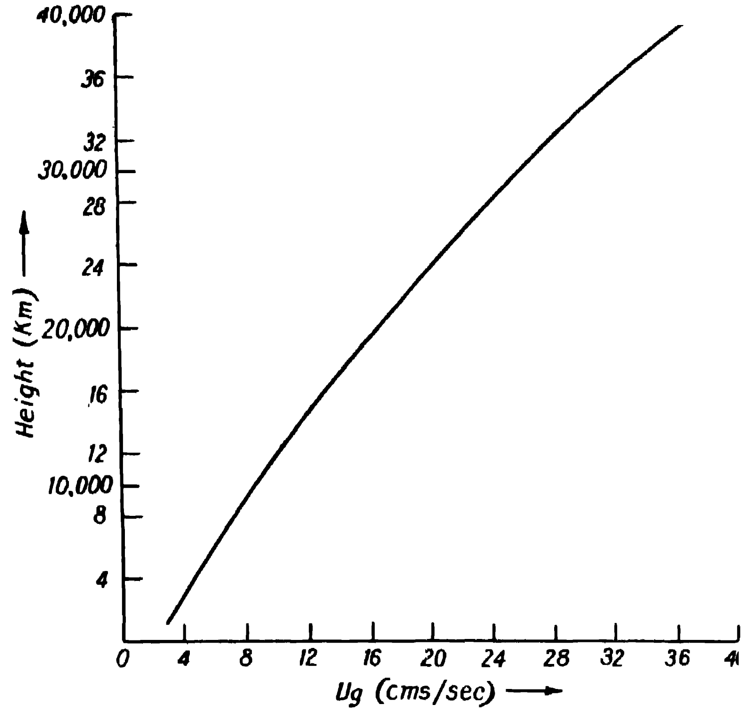

FIGURE 6

§4. ROTATION OF THE EARTH MAGNET—TIME OF ARRIVAL OF THE IONS IN HIGH LATITUDES

In the above discussion, the tacit assumption has been made that the earth is not rotating. If, however, the earth's rotation is taken into account, then the motion of the lines of force causes certain complications in the motion of the ion, which we will now consider. It is obvious in the first place that uncharged particles will not participate in the peripheral motion of the earth's atmosphere because by our assumption the particles in the " spray " are moving in the region of no collision and their motion is determined only by the initial velocities acquired by them during the last super-elastic collision. This is not so for charged particles, that is, for ions and electrons. Such particles in their cycloidal motion round the magnetic lines of force will be subject to the electric force which is developed due to the motion of the magnetic lines of force of the rotating earth magnet. In order to determine how the motion will be affected, we start with the well-known Lorentz equation

$$E = \frac{[v \times H]}{c} \tag{22}$$

where E is the electric field generated by the motion of the magnetic field H with velocity v.

Since the earth may be regarded as a magnet rotating round its axis (neglecting the inclination of the geographical to the magnetic axis) at any point above the surface of the earth, there will be developed an electric field of intensity given by the above equation. It may be shown, after Swann,[25] that the rotation of the earth-magnet attributes an electric polarisation $P = \frac{[(\Omega \times r) \times I]}{c}$ to it. For points outside the earth's surface, this gives rise to an electric field having components [26]

$$\left.\begin{aligned} E_r &= \frac{M\Omega}{4\pi c a^2} \frac{a^4}{r^4} (1 - 3\cos^2\theta) \\ E_\theta &= -\frac{2M\Omega}{4\pi c a^2} \frac{a^4}{r^4} \sin\theta\cos\theta \end{aligned}\right\} \qquad \cdots (23)$$

where θ is the angle made by r with the axis of the earth, and obviously the magnetic field has components

$$\left.\begin{aligned} H_r &= \frac{2M}{4\pi r^3} \cdot \cos\theta \\ H_\theta &= \frac{M}{4\pi r^3} \cdot \sin\theta \end{aligned}\right\} \qquad \cdots (24)$$

The motion of the ion in this crossed electro-magnetic field will, therefore, impart a drift velocity to the ion given by

$$v' = \frac{c[E \times H]}{H^2}$$

$$= (\vec{\Omega}\vec{r}) \frac{a^2}{r^2} \frac{1 + \cos^2\theta}{1 + 3\cos^2\theta} \qquad \cdots (25)$$

Eqn. (25) shows that when $r >> a$, $v' \to 0$ and for $r = a$, and $\theta = 90°$, $v' = \Omega a$, *i.e.*, rotational velocity of the earth. In other words, an ion at a distance of several earth-radii from the earth will move spiralling round the magnetic line of force of a nearly stationary earth, but as it descends, it will gradually experience a drag as if it is beginning to participate in the earth's rotation. This drift velocity will have an important effect on the final position of the arrival of the ion near the surface of the earth. If the ion participated in the full velocity of the rotation of the earth, then the ion could have glided down a particular line of force and would have arrived at the meridian on which the line of force in question strikes the earth. If, however, we assume that the ion is not participating in the rotation of the earth, then the particle would arrive at a meridian $\Phi = \left(\frac{360 \times t}{24 \times 60 \times 60}\right)^\circ$ west of the meridian at which the line of force strikes a non-rotating earth. If, however, the ion participates only partly in the peripheral velocity of the earth, then it will arrive at a meridian which is intermediate between the above two. It is easy to calculate the amount of the shift, *i.e.*, to calculate at what local hour an ion would arrive at the surface of the earth if we know the hour at which it was ionized and also its speed at the moment of ionization

The eastward velocity experienced by an ion at a distance r from the earth's centre due to the rotation of the earth magnet is

$$(\vec{\Omega}_1)\frac{a^2}{r^2}\frac{1+\cos^2\theta}{1+3\cos^2\theta}$$

so that the angular velocity at the point under consideration is

$$\Omega.\frac{a^2}{r^2}\frac{1+\cos^2\theta}{1+3\cos^2\theta}$$

Obviously this angular velocity varies at each point of the downward course of the ion as r changes, so that the total angle turned through is

$$\int_{t_1}^{t_2}\frac{\Omega a^2}{r^2}\frac{1+\cos^2\theta}{1+3\cos^2\theta}.dt = \int_{s_1}^{s_2}\frac{\Omega a^2}{r^2}\frac{1+\cos^2\theta}{1+3\cos^2\theta}\frac{ds}{v}, \qquad \ldots (26)$$

where (t_2-t_1) is the time of descent along the magnetic line of force, and v is the velocity along the magnetic line of forces. Knowing the equation of a magnetic line of force

$$\frac{\sin^2\theta}{\sin^2\theta_0} = \frac{a}{r}$$

the above integration can be performed graphically.

Φ, the total angle turned through, being known, it is easy to calculate at what local hour the ion will return to the earth's atmosphere. This is given by

$$t = \frac{24 \times 60 \times 60}{360^\circ} \times \Phi^\circ \text{ secs.}$$

It is found by calculation that an atom ionized at 12 o'clock at 40,000 km. at equator and descending along the magnetic line of force with 1 km./sec. velocity will return to the atmosphere at about 12-54 local hour, *i.e.*, about 54 minutes behind. If the atom be ionized at 14,000 km. it will reach the top of the atmosphere at about 12-30 P.M.

THE AVERAGE PATH OF AN ION

We may now discuss the nature of the average path followed by an ion as it proceeds from the equator to higher latitudes due to the combined effect of the following velocities :

(1) Velocity along the magnetic line of force of magnitude v.

(2) Gravitational-magnetic drift velocity u_g at right angles to g and H of magnitude $\frac{mg}{Hc}\,.\,c$ (opposite direction for electrons).

(3) Velocity v' due to rotation effect of the earth-magnet of magnitude $\Omega.\frac{a^2}{r}\frac{1+\cos^2\theta}{1+3\cos^2\theta}$ in the direction of rotation

where r=distance from the centre of the earth;

θ=angle made by r with the axis of the earth;

g=acceleration due to gravity at the point under consideration;

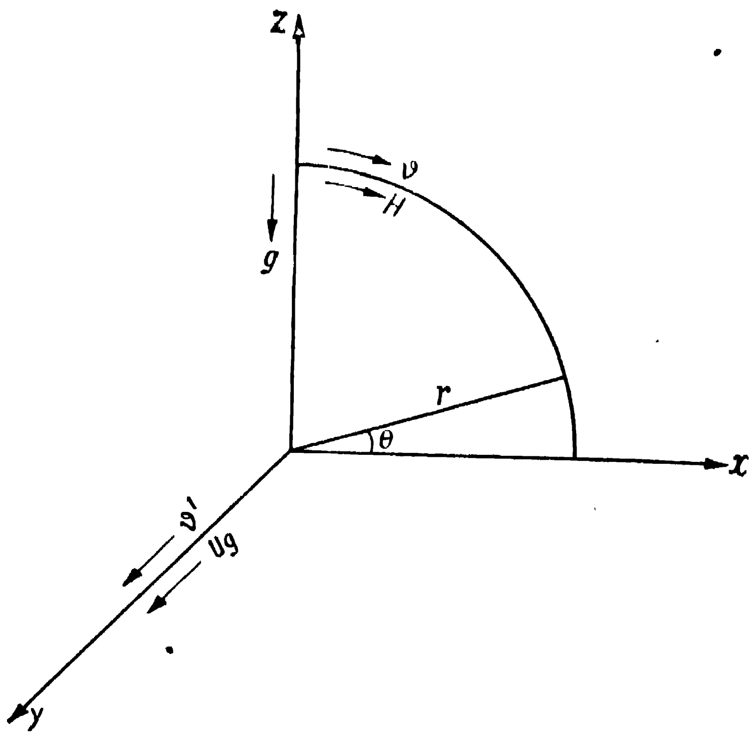

FIGURE 7

a = radius of the earth ;

H = magnetic field of the earth at the point under consideration;

Ω = angular velocity of the rotation of the earth.

(The component velocities are shown in Figure 7.)

The velocity v has components $v.\sin\theta$ and $-v.\cos\theta$ along x and z axes respectively. Thus the velocity components for an ion are

$$\left.\begin{aligned} \dot{x} &= v\sin\theta \\ \dot{y} &= c\frac{mg_o}{cH_o a}\cdot\frac{r}{1+\cos^2\theta}+\Omega\frac{a^2}{r}\frac{1+\cos^2\theta}{1+3\cos^2\theta} \\ \dot{z} &= -v\cos\theta \end{aligned}\right\} \quad \ldots \quad (27)$$

$$\because \qquad g = g_o\frac{a^2}{r^2}$$

$$H = H_o\frac{a^3}{r^3}(1+\cos^2\theta)$$

or the velocity of the ion at the point under consideration is

$$\begin{aligned} V &= (\dot{x}^2+\dot{y}^2+\dot{z}^2)^{\frac{1}{2}} \\ &= \left[v^2+\left\{c\frac{mg_o}{cH_o a}\cdot\frac{r}{1+\cos^2\theta}+\Omega\frac{a^2}{r}\frac{1+\cos^2\theta}{1+3\cos^2\theta}\right\}^2\right]^{\frac{1}{2}} \end{aligned} \quad \ldots\ldots \quad (28)$$

In the above expression, v is constant for all altitudes and of the other two terms $c\frac{mg_o}{cH_o a}\cdot\frac{r}{1+\cos^2\theta}$ and $\Omega\frac{a^2}{r}\frac{1+\cos^2\theta}{1+3^2\cos\theta}$ the former represents the effect of the gravitational-magnetic drift velocity and the latter that of rotation of the earth-magnet. When r is very much larger than a, the second term becomes very small, in other words, the rotation of the magnetic field is negligible. If r is comparable with a, the second term becomes much greater than the first and the effect of rotation of the magnetic field begins to be appreciable. Thus at very high altitudes the ions will proceed along a magnetic line of force due as it were to a non-rotating earth, as the ions come to lower levels it begins to participate in the lateral motion of the lines of force and are carried with them in the direction of rotation of the earth.

Now the range of heights under consideration does not exceed 7 or 8 times the radius of the earth. It is easily seen that within this height the second term is always much greater than the first. In fact as we come downwards from higher levels, the second term rapidly increases in value while the first term becomes small in comparison with the second. This means that the ions have always a tendency to participate in the transverse motion of the earth. This tendency increases as the ions come down and at heights of a few hundred kilometres from the earth the electrons and ions move with the earth as if they form a part of the outer atmosphere.

§5 THE ULTRA-VIOLET LIGHT THEORY OF AURORAE AND MAGNETIC DISTURBANCES

In the preceding sections we have studied the state of affairs at the farthest limits of the atmosphere on the basis of our most recent knowledge of the constituents and temperature of the upper atmosphere. We have found that the fringe region or region of free flight of the atmospheric particles begins from a height of about 800 km. from the earth's surface. Obviously the lightest constituent of the gases of the upper atmosphere will predominate in number in the fringe region. Hydrogen and helium being absent, as is now believed, the fringe will contain oxygen in the atomic state. The question of dissociation of nitrogen in the upper atmosphere is not yet definitely settled. We will now see how the present study enables one to test quantitatively the current ultra-violet light theory of aurorae and magnetic storms.

Firstly, we have seen that the highest energy available by a super-elastic collision is only 4·2 electron volts which can impart a velocity of about 7 km.-sec. to an oxygen atom. This velocity enables a particle to attain a height of about 14,000 km. above the earth's centre. We do not, therefore, find particles speedy enough to reach a height of 40,000 km. or so, as is demanded by the ultra-violet light theory to account for the maximum frequency of occurrence of aurorae at 67° latitude.

A remarkable feature of the auroral spectrum is that the intensity of the green line of oxygen and the negative bands of nitrogen are of the same order whereas in the spectrum of the night sky the nitrogen bands are very feeble. The ultra-violet light theory attempts to give an explanation of this on the supposition that neutral nitrogen molecules from the equatorial region are transported to high levels in the fringe region and by being ionized are thence guided to the polar regions; unfortunately, however, nitrogen, if it exists in the molecular state, will be confined far below the level of escape and will have, therefore, little chance of being transported from the equatorial to the polar region.

The number of high-speed particles thus crossing each sq. cm. area of the top of the atmosphere at the equatorial region and again entering the atmosphere at the polar region, as calculated in the present paper, is only 10^4

at the maximum; this is less by about three orders than the number assumed in the ultra-violet light theory of Hulburt.[2]

Coming now to the question of ionization of high-flying particles in the fringe region, here too we find some difficulty. The time for such a high-flying atom to get ionized by the ultra-violet rays of the sun is given by

$$t = \frac{E}{I_0(1-e^{-\tau_0})} \tag{29}$$

where E = Energy required to ionize the atom ;
I_0 = Intensity of ionizing solar ultra-violet radiation ;
τ_0 = Atomic absorption co-efficient.

We have as yet no experimental value of τ_0 for the process $O^3P \rightarrow O^{+\,4}S$ but taking the theoretical value of Saha and Rai, $\tau_0 = 2{\cdot}81 \times 10^{17}$, we find it to be much less than the average value (3×10^{11}) assumed by Hulburt and Maris. The intensity of ultra-violet solar radiation within the limits $\lambda = 912$Å to $\lambda = 0$, assuming the sun to be a black body radiator even in the ultra-violet portion is about $6{\cdot}6 \times 10^{2}$ erg-sec.-cm.[2] It can easily be calculated that the time required for an oxygen atom to be ionized is about 3×10^2 hours. This is about a thousand times greater than the time of ionization as calculated by Hulburt. In other words, this would lead us to the conclusion that the oxygen atoms in their path in the fringe region have very little chance of being ionized. We may mention here, however, that all such calculations cannot be regarded as final since the extreme ultra-violet portion of the solar spectrum departs widely from that of a black-body radiator at 6800°K and is most probably of the nature of line emission.

McNish[27] in a recent paper has pointed out that on three occasions bright solar eruptions were not accompanied by magnetic storms and regards this as irreconcilable with the ultra-violet light theory. This argument against ultra-violet light theory requires, however, careful re-examination since the bright solar eruptions due to the intensification of a particular line does not necessarily mean enhancement of the ionizing radiation required to ionize the particles in the fringe region.

§6. SUMMARY AND CONCLUSION

The discussion and the analysis given in the previous sections show that the atmosphere may extend up to great heights if its fringe or the spray region is taken into account. The spray region begins from a level at which the molecules begin to experience negligible collision with one another. If, as is now usually believed, the higher regions of the atmosphere consist of atomic oxygen and is at a high temperature (1000°K), then it is found that the transition from the region of collision to region of no collisions will occur between heights 770 km. and 930 km. The oxygen atoms escaping from this region and describing orbits—elliptic, hyperbolic or parabolic—round the earth will, in the absence of any disturbing effect, form the spray region of the earth's atmosphere. (The atoms with hyperbolic orbits will, of course, escape altogether from the earth's atmosphere.) The average density of atoms in the spray region will rapidly diminish with increasing height and its value will approach that of the interstellar space, *viz.*, about one particle per c.c. at a height of 3000 km. from the surface of the earth.

If account is taken of super-elastic collisions which some of the atoms might suffer in the region of escape (770-930 km.) due to atoms or molecules in this region being excited to metastable states by the incident solar radiation, then the spray region is found to extend to the height of 14,000 km

These results are at variance with the supposition which Hulburt makes for explaining aurora and magnetic storms. According to Hulburt super-elastic collisions might drive particles to heights of about 42,000 km. A critical examination of the possible modes of super-elastic collisions, in the light of the available spectroscopic knowledge, does not warrant the existence of particles speedy enough to reach such enormous heights.

We have seen that, in the highly rarefied atmosphere we are considering, only those atoms which can remain in the excited states for a sufficiently long time (*i.e.*, atoms and molecules in the metastable states) can have a chance of imparting high velocities to other particles. The metastable states which can effect this are the ^1S for oxygen atom and the $A^3\Sigma_u^+$ state of the nitrogen molecules. The energies of these states are, however, 4.2 ev and 6.15 ev and the velocity imparted by them to an oxygen atom are 7·1 km./sec. and 8·6 km./sec. respectively. Hulburt's particles reaching 42,000 km. height must receive sufficient energy to attain velocity of the order of 10 km./sec. (Note—At great altitudes a small excess of velocity takes the particle a long way up since g is very small at high levels.) Our present spectroscopic knowledge of the upper atmosphere does not indicate any source of metastable atoms with such high energy.

If the ionization of the particles thrown up by super-elastic collisions is taken into account, it is found that the ions will be entangled in the magnetic field of the earth and the combined action of the gravity and the magnetic field will lead them along magnetic lines of force from the equatorial to higher latitudes. Here again the average time (3×10^3 hours) taken for ionization of the particles calculated from the theoretical value of the absorption coefficient of atomic oxygen ($\tau_0 = 2{\cdot}81 \times 10^{-17}$ for the ionizing wavelength) is found to differ widely from that assumed by Hulburt.[2] Hulburt's particles are assumed to take about 3 hours for ionization while the calculation referred to above yields results which are about three orders higher than Hulburt's value. In order that Hulburt's hypothesis of the production of aurorae and magnetic storms by the import of ions transferred from low to high latitudes be applicable, it is necessary to have atoms or molecules with absorption coefficients at least three orders higher than that of atomic oxygen and/or to have the ionizing radiation in the portion of the solar spectrum under consideration greater by the same amount than that calculated from black-body solar radiation.

In connection with the motion of the ions in the rotating magnetic field of the earth an interesting point arises. How will the lateral motion of the magnetic field of the rotating earth-magnet affect the motion of the ion? It is seen that this motion of the magnetic field will produce an electric field according to the Lorentz equation $E = \frac{[v \times H]}{c}$ and that the field will have the effect of dragging the ions along with it round the earth; in other words, the ions in the spray region, unlike neutral particles, will partially participate in the rotation of the earth. The rotation effect at a height r will be $\left[\frac{a^2}{r^2} \frac{1+\cos^2\theta}{1+3\cos^2\theta}\right]$ of the rotational velocity of the lines of force at that height. As the surface of the earth is approached, the dragging effect will increase and the rotational velocity of the ion will also approach the rotational velocity of the earth.

An important consequence of the entanglement of the ions with the magnetic lines of force will be the reduction of the rate of escape of molecules from the outer atmosphere of the earth. A neutral particle in the equatorial region having a velocity 11 km./sec. at a height 7000 km. from the earth's centre will escape from the attraction of the earth. The same particle, if ionized, will require a velocity of $2 \cdot 5 \times 10^5$ km./sec. in order to escape by disentangling itself from the earth's magnetic field.

A rough quantitative estimate of the various factors regarding the formation and ionization of the particles in the spray region shows that the assumptions underlying the ultra-violet theory of aurora and magnetic storms as postulated by Hulburt [2] do not show very satisfactory quantitative agreement with the available data of the high atmosphere. One may, however, expect that with the increase of our spectroscopic knowledge regarding the constituents of the high atmosphere new facts may emerge which would show the way out of the difficulty and would explain the discrepancies discussed in the paper.

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